Power functions $y = ax^n$ where n is a simple rational number, and their graphs.

Power Functions

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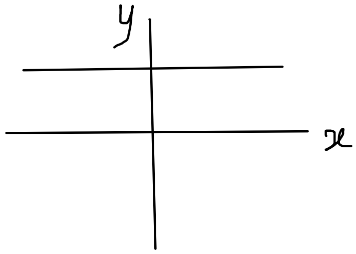
e.g. sketch the graphs of :

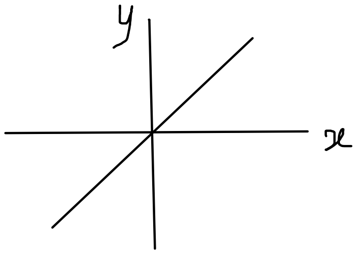
$y = 1$

x	-2	-1	0	1	2
y	1	1	1	1	1



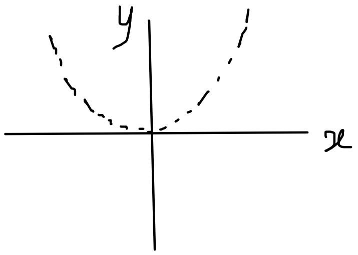
$y = x$

x	-2	-1	0	1	2
y	-2	-1	0	1	2



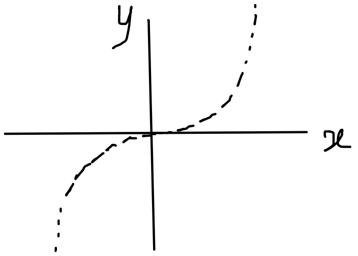
$y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4



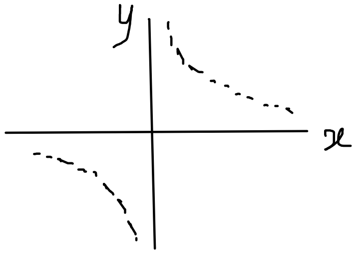
$y = x^3$

x	-2	-1	0	1	2
y	-8	-1	0	1	8



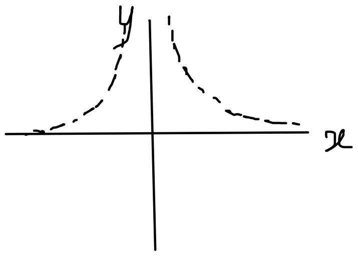
$y = x^{-1}$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$-\frac{1}{2}$	-1	-2	2	1	2



$y = x^{-2}$

x	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y	$\frac{1}{4}$	1	4	4	1	$\frac{1}{4}$



Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs.

Exponential Functions

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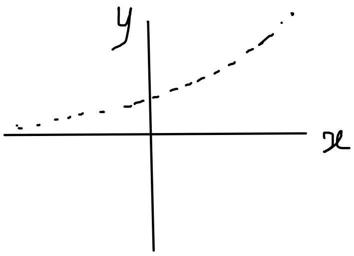
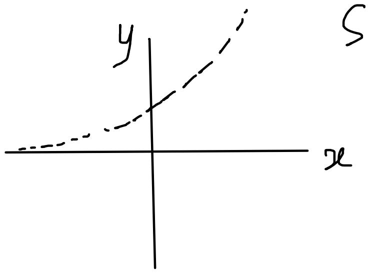
e.g. Sketch $y = 2^x$.

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



e.g. Sketch $y = e^x$. $e = 2.71828\ldots$

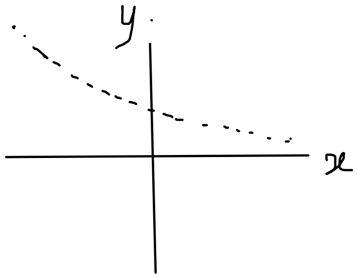
x	-2	-1	0	1	2
y	0.14	0.37	1	2.7	7.4



Similar

e.g. Sketch $y = 2^{-x}$

$y = f(-x)$ reflect in y axis



e.g. Sketch $y = e^{-x}$

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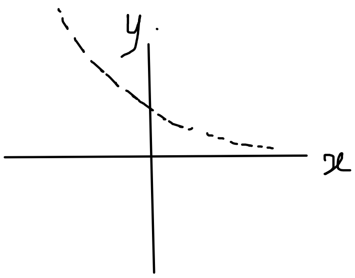

Exponential and logarithmic functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs.

Logarithmic Functions

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e.g. $y = \log_{10} 100$ means $10^y = 100$.

So log is the inverse of power function.

$$\underset{\text{base}}{y = \log_a x} \iff \overset{\text{power}}{\underset{\text{base}}{a^y}} = x$$

$\therefore \log_{10} 100 = 2$ because $100 = 10^2$.

e.g. $y = \log_e 5$ means $e^y = 5$.
$(e = 2.71828\ldots)$

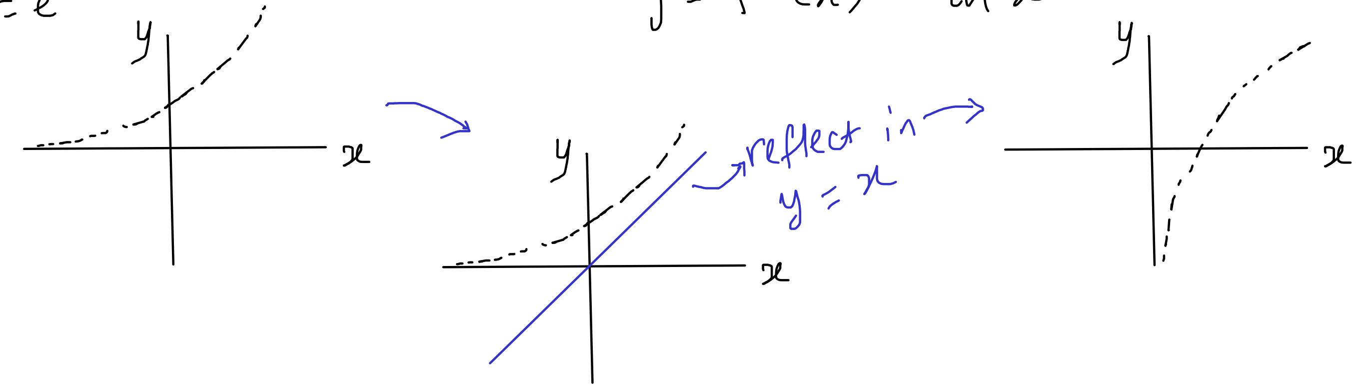
Notation: $\log_e 5 = \ln 5$ (natural log)

e.g. Sketch graph of $y = \ln x$.

$y = \log_e x \Rightarrow e^y = x$, inverse of $y = e^x$.

$y = f(x) = e^x$

$y = f^{-1}(x) = \ln x$



Laws of Logarithm

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Identities $\boxed{\log_b 1 = 0}$, if $b > 0$ e.g. $\log_2 1 = 0$ $\because$ $1 = 2^0$

$\boxed{\log_b b = 1}$ $\log_2 2 = 1$ $\because$ $2 = 2^1$

$\boxed{\log_b b^x = x}$ $\log_2 2^3 = 3$ $\because$ $2^3 = 2^3$

$\boxed{b^{\log_b x} = x}$ $2^{\log_2 3} = 3$ $\because$ $\log_2 3 = \log_2 3$

Laws $\boxed{\log_b xy = \log_b x + \log_b y}$ $\log_2(3\times 4) = \log_2 3 + \log_2 4$

$$\because \; 3\times 4 = 2^{\log_2 3 + \log_2 4} = 2^{\log_2 3} \times 2^{\log_2 4} = 3 \times 4$$

$\boxed{\log_b \frac{x}{y} = \log_b x - \log_b y}$

$\boxed{\log_b x^a = a \log_b x}$ $\log_2 3^4 = 4 \log_2 3$

$$\because \; 3^4 = 2^{4\log_2 3} = (2^{\log_2 3})^4 = 3^4$$

Changing base $\boxed{\frac{\log_d a}{\log_d b} = \log_b a}$ e.g. $\frac{\log_4 3}{\log_4 2} = \log_2 3$

$\because$ $\curvearrowleft$ $\log_4 3 = \log_4 2 \log_2 3$

$\curvearrowleft$ $3 = 4^{\log_4 2 \log_2 3}$

$\curvearrowleft$ $3 = 2^{\log_2 3}$

If $d = a$,

$$\frac{\log_a a}{\log_a b} = \log_b a$$

Get $\boxed{\frac{1}{\log_a b} = \log_b a}$ identity

Modulus functions $|x|$ and $|f(x)|$ where $f(x)$ is linear, quadratic or trigonometric, and their graphs.

Modulus Functions

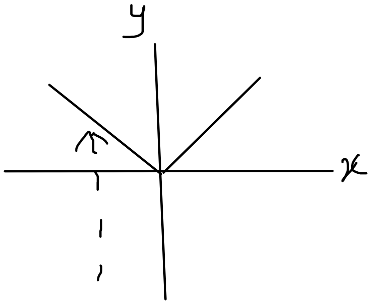
Dr. K. M. Hock

$|x|$ means we take the +ve value.

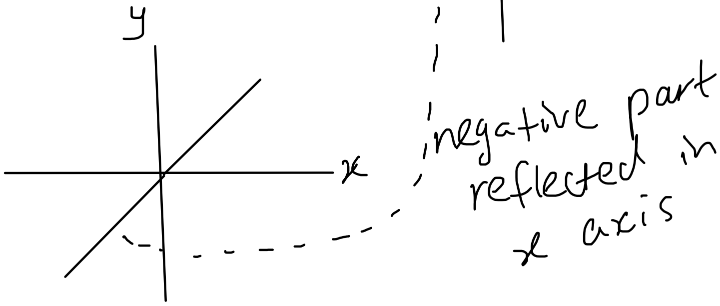
e.g. $|-5| = 5$, $|3| = 3$, $|-0.2| = 0.2$

e.g. Sketch graph of $y = |x|$

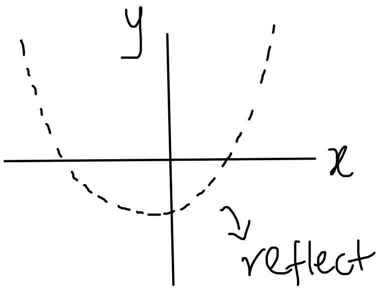
x	-2	-1	0	1	2
y	2	1	0	1	2



Compare with $y = x$



e.g. given $y = f(x)$ graph, sketch $y = |f(x)|$.



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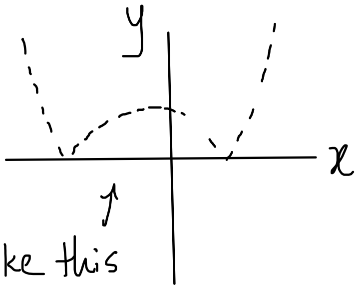

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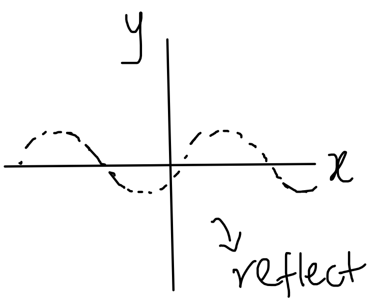

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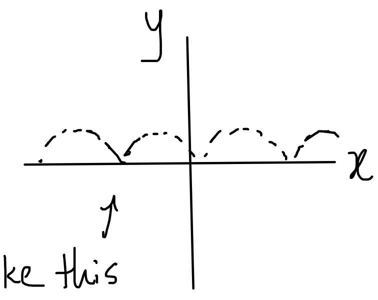


Problem 1

Dr. K.M. Hock

2012 P2 Q6 (a)(i) Given that $\log_8 x^3 = \log_4 u$, express u in terms of x.

(ii) Find the value of x for which $\log_4 (x^2+5x) - \log_8 x^3 = \frac{1}{\log_3 4}$

(b) Solve the equation $e^y(e^y-2) = 15$.

Solution. (a)(i)

$$\log_8 x^3 = \frac{\log_8 u}{\log_8 4}$$

$4 = 2^2$, $8 = 2^3$

$\longleftarrow$ So $4 = 8^{2/3}$

$$= \frac{\log_8 u}{2/3}$$

$$\log_8 u = \frac{2}{3}\log_8 x^3 = \log_8 (x^3)^{2/3} = \log_8 x^2$$

$$\therefore u = x^2$$

(ii)

$$\log_4 (x^2+5x) - \log_8 x^3 = \frac{1}{\log_3 4}$$

$$\log_4 (x^2+5x) - \log_4 u = \log_4 3$$

$$\log_4 (x^2+5x) - \log_4 x^2 = \log_4 3$$

$$\log_4 (x^2+5x) = \log_4 3 + \log_4 x^2$$

$$\log_4 (x^2+5x) = \log_4 3x^2$$

$$x^2 + 5x = 3x^2$$

$$0 = 2x^2 - 5x$$

$$0 = x(2x-5)$$

$$x = 0, \frac{5}{2}$$

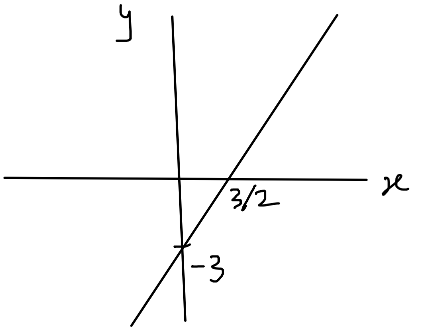
Problem 2

Dr. K.M. Hock

2013 P1 Q1 Calculate the co-ordinates of the points of intersection of the graph of $y = |2x-3| - 2$ with the coordinate axes.

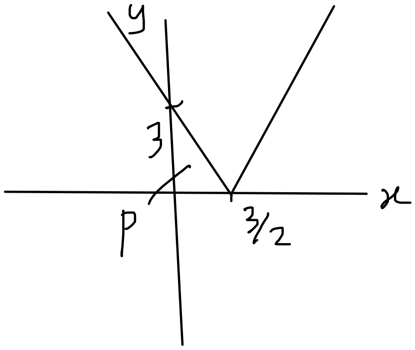
Solution. Let $y = 2x-3$

Find intercepts, sketch →



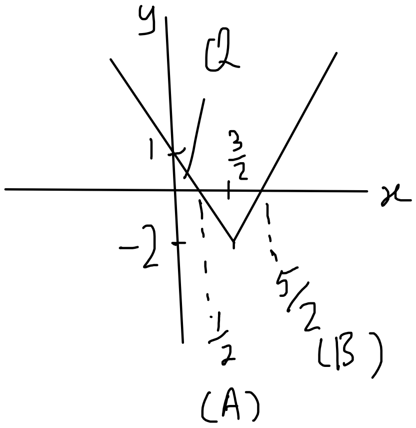
Let $y = |2x-3|$

Reflect →



Let $y = |2x-3| - 2$

Move down →



Use similar Δ's P & Q, get intercept A.

Use symmetry about $x = \frac{3}{2}$, get intercept B.

Problem 3

Dr. K.M. Hock

2013 P1 Q8 The temperature, T °C, of a chicken removed from a freezer is given by the formula $T = 20 - 38e^{-0.6t}$, where t is the time in hours since the chicken was removed from the freezer.

(i) Find the temperature at which the chicken was kept in the freezer.

(ii) Find the temperature of the chicken when $t = 2$.

(iii) Express t as a function of T.

(iv) Explain why the temperature of the chicken can never reach 20 °C.

Solution.

(i). $t = 0$, $T = 20 - 38 = -18$ °C.

(ii). $T = 20 - 38e^{-0.6(2)} = 8.555$ °C

(iii)

$$T = 20 - 38e^{-0.6t}$$

$$38e^{-0.6t} = 20 - T$$

$$e^{-0.6t} = \frac{20-T}{38}$$

$$-0.6t = \ln \frac{20-T}{38}$$

$$t = -\frac{1}{0.6} \ln \frac{20-T}{38}$$

(iv) Because $e^{-0.6t}$ can never reach 0.

It can only get smaller as t gets larger.